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I-1045

M.A./M.Sc. (Previous) Examination, 2020

MATHEMATICS

Paper - II

(Real Analysis and Measure Theory)

Time Allowed : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 36

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. (a) Prove that $f \in R(\alpha)$ on [a, b] if and only if for revery $\in > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$.

(b) If $f \in R$ on [a, b] and if there is a differentiable function F on [a, b] such that $F' = f$, then prove that $\int_a^b f(x)dx = F(b) - F(a)$.

Q. 2. Assume α increases monotonically and $\alpha' \in R$ on [a, b]. Let f be a bounded real function on [a, b]. Then prove that $f \in R(\alpha)$ if and only if $f\alpha' \in R$.

Q. 3. (a) Suppose $\{f_n\}$ is a sequence of functions defined on E, and suppose $|f_n(x)| \le M_n$ ($x \in E$, n = 1, 2, 3, ...), then prove that Σf_n converges uniformly on E if ΣM_n converges.

(b) If K is a compact metric space, if $f_n \in \mathscr{C}(k)$ for n = 1, 2, 3, ... and if $\{f_n\}$ converges uniformly on K, then prove that the $\{f_n\}$ is equicontinuous on K.

Q. 4. Let $\{f_n\}$ is a sequence of functions, differentiable on [a, b] and such that $\{f_n(x_0)\}$ converges for some point x_0 on [a, b]. If $\{f_n'\}$ converges uniformly on [a, b], then prove that the $\{f_n\}$ converges uniformly on [a, b] to a function f and :

$$f'(x) = \lim_{n\to\infty} f_n'(x) \quad (a \le x \le b)$$

Q. 5. (a) Prove that every uniformly sequence of bounded functions is uniformly bounded.

(b) State and prove Abel's theorem.

Q. 6. Let $\sum C_n$ converges. Put $f(x) = \sum_{n=0}^{\infty} C_n x^n \ (-1 < x < 1)$ then prove that :

$$\lim_{x\to 1} f(x) = \sum_{n=0}^{\infty} C_n$$

Q. 7. (a) Prove that a linear operator A on a finite dimensional vector space X is one to one if and only if the range of A is all of X.

(b) Let f maps on open set $E \subset R^n$ into R^m. Then prove that $f \in \mathscr{C}'(E)$ if and only if the partial

derivatives $D_j f_i$ exist and are continuous on E for $1 \le i \le m$, $1 \le j \le n$.

Q. 8. State and prove Stoke's theorems.

Q. 9. State and prove Lebesgue's dominated convergence theorem.

Q. 10. (a) Prove that BA is linear if A and B are linear transformations.

(b) If f is a differentiable mapping of a connected open set $E \subset R^n$ into R^m and if $f'(x) = 0$ for every $x \in E$. Prove that f is constant in E.