I-1046

M.A./M.Sc. (Previous) Examination, 2020

MATHEMATICS

Paper - III

(Topology)

Time Allowed : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 36

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. (a) If $\mathfrak{J}_1$ and $\mathfrak{J}_2$ be two topologies defined for a non-empty set X, then prove that $\mathfrak{J}_1 \cap \mathfrak{J}_2$ is also a topology for X.

(b) Let (X, $\mathfrak{J}$) be a topological space and A and B be any two subsets of X. If $\overline{A}$ denotes the closure of A then prove that :

(i) $A \subset B \Rightarrow \overline{A} \subset \overline{B}$

(ii) $\overline{A \cup B} = \overline{A} \cup \overline{B}$

Q. 2. (a) Prove that the continuous image of a compact space is compact.

(b) Prove that a second countable is always first countable space.

Q. 3. (a) Define normal & complete normal space. And also prove that every compact Hausdorff space is normal.

(b) State & prove Urysohn's lemma.

Q. 4. State & prove Tietze-extension theorem.

Q. 5. (a) Define compactness. Prove that every compact spaces has Bolzano-Weierstrass property.

(b) Prove that a compact space is locally compact but not conversely.

Q. 6. (a) Define component of a space. Prove that every component of a topological space is closed.

(b) Prove that a closed subspace of a Lindeloff space is a Lindeloff space.

Q. 7. (a) Prove that a topological space X is locally connected iff the components of every open subspace of X are open in X.

(b) Prove that a topological space $(X, \mathfrak{J})$ is disconnected iff $\exists$ a non empty proper subset of X which is both $\mathcal{J}$-open and $\mathcal{J}$-closed in X.

Q. 8. State & prove Urysohn's metrization theorem.

Q. 9. (a) Prove that a topological space $(X, \mathfrak{J})$ is Hausdorff iff every net in X can converge to atmost one point.

(b) Prove that if $(X, \mathfrak{J})$ be a topological space and $Y \subset X$, then a point $x_0 \in X$ is a limit point of Y iff $\exists$ a net in $Y - \{x_0\}$ converges to $\{x_0\}$.

Q. 10. (a) Prove that every filter F on set X is the intersection of all ultrafilters finer than F.

(b) Prove that every filter is contained in an ultrafilter.