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I-1059

M.A./M.Sc. (Final) Examination, 2020

MATHEMATICS

Paper - VII

(Fuzzy Sets and Their Application)

Time Allowed : Three Hours

Maximum Marks : 100

Minimum Pass Marks : 36

Note : Attempt any five questions. All questions carry equal marks.

Q. 1. Define following terms with example (any five) :

(i) Fuzzy sets

(ii) α-cut of a fuzzy set

(iii) Cutworthy and strong cutworthy property

(iv) Standard fuzzy complement

(v) Normal and subnormal fuzzy sets

(vi) Degree of subsethood

Q. 2. (a) State and prove second decomposition theorem.

(b) Let f : X $\rightarrow$ Y be an arbitrary crisp function. Then prove that, for any A $\in$ F(X), f fuzzified by the extension principle satisfies the equation :

$$f(A) = \bigcup_{\alpha \in [0,1]} f({}_{\alpha+}A)$$

Q. 3. (a) If C is continuous fuzzy complement then show that C has a unique equilibrium.

(b) Let i be a t-norm and C be an involutive fuzzy complement. Then show that the binary operation U on [0, 1] defined by :

U(a, b) = C (i(c(a), c(b)))

is a t-conorm.

Q. 4. State and prove first characterization theorem for fuzzy complement.

Q. 5. (a) Solve the fuzzy equation

A + X = B

if $A = \frac{\cdot 2}{[0,1)} + \frac{\cdot 6}{[1,2)} + \frac{\cdot 8}{[2,3)} + \frac{\cdot 9}{[3,4)} + \frac{1}{4} + \frac{\cdot 5}{(4,5]} + \frac{\cdot 1}{(5,6]}$

$B = \frac{\cdot 1}{[0,1)} + \frac{\cdot 2}{[1,2)} + \frac{\cdot 6}{[2,3)} + \frac{\cdot 7}{[3,4)} + \frac{\cdot 8}{[4,5)} + \frac{\cdot 9}{[5,6)} + \frac{1}{6} + \frac{\cdot 5}{(6,7]} + \frac{\cdot 4}{(7,8]} + \frac{\cdot 2}{(8,9]} + \frac{\cdot 1}{(9,10]}$

(b) Solve the fuzzy equation

A·X = B

if $A(x) = \begin{cases} 0 & \text{for} \quad x \le 3 \text{ and } x > 5 \\ x - 3 & \text{for} \quad 3 < x \le 4 \\ 5 - x & \text{for} \quad 4 < x \le 5 \end{cases}$

$B(x) = \begin{cases} 0 & \text{for} \quad x \le 12, x > 32 \\ \frac{x - 12}{8} & \text{for} \quad 12 < x \le 20 \\ \frac{32 - x}{12} & \text{for} \quad 20 < x \le 32 \end{cases}$

Q. 6. (a) Check whether the following relation :

$$R(X,X)=\begin{array}{c|cccc|} & a & b & c & d \\ a & 1 & \cdot 8 & 0 & \cdot 4 \\ b & \cdot 8 & 1 & 0 & \cdot 4 \\ c & 0 & 0 & 1 & 0 \\ d & \cdot 4 & \cdot 4 & 0 & 1 \end{array}$$

is an equivalence relation, where X = {a, b, c, d}.

(b) Write short notes on fuzzy morphism.

Q. 7. (a) If R is any fuzzy relation on X^2 then prove that the fuzzy relation

$$R_{T(i)} = \bigcup_{n=1}^{\infty} R^{(n)}$$

is the i-transitive closure of R.

(b) For any a, a_j, b, d $\in$ [0, 1], show that :

(i) $i(a, b) \leq d$ iff $w_i(a, d) \geq b$

(ii) $w_i\left[\sup_{j\in J} a_j, b\right] = \inf_{j\in J} w_i(a_j, b)$

Q. 8. (a) Show that every possibility measure Pos on a finite power set P(X) is uniquely determined by a possibility distribution function :

$r : X \to [0, 1]$ via the formula

$$\text{Pos}(A) = \max_{x\in A} r(x)$$

for each A $\in$ P(X)

(b) Prove that a belief measure Bel on a finite power set P(X) is a probability measure if and only if the associated basic probability assignment function m is given by m({x}) = Bel ({x}) and m(A) = 0 for all subsets of X that are not singletons.

Q. 9. Write short notes on fuzzy quantifiers.

Q. 10. Write short notes on :

(i) Multiperson decision making

(ii) Fuzzy linear programming