M-5342

M.A./M.Sc. (IVth Semester) Examination, 2020

MATHEMATICS

(Partial Differential Equations and Mechanics)

Time Allowed : Three Hours

Maximum Marks : 70

Note : Attempt all the following questions. Each question carries 14 marks.

Q. 1. Obtain the complete integral of the differential equation :

$$p^3x + q^3y = 0$$

OR

Solve the following PDE, $xU_x + yU_y + UU_z = 0$ subject to the condition $U(x, y, 0) = xy$, where both $x > 0$ and $y > 0$, using the method of characteristics.

Q. 2. Find the bounded solution of :

$$\frac{\partial y}{\partial x} = 2\frac{\partial y}{\partial t} + y$$

if $y(x, 0) = 6e^{-3x}$

OR

Find out the solution of :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

by the method of separation of variables.

Q. 3. State and prove Cauchy-Kovalevskaya theorem.

OR

Show that the general solution of one dimensional wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

is $u = f(x - ct) + \phi(x + ct)$

Q. 4. Show that :

$$P = \frac{1}{2}(p^2 + q^2),$$

$$Q = \tan^{-1}\left(\frac{q}{p}\right)$$

is canonical.

OR

State and prove Lee Hwa Chung's theorem.

Q. 5. Find the relation between Poisson brackets and Lagrange bracket.

OR

State and prove Jacobi theorem.

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